LIE-ISOTOPIC FINSLERIAN LIFTING OF THE LORENTZ GROUP AND BLOKHINTSEV-REDEI-LIKE BEHAVIOR OF THE MESON LIFE-TIMES AND OF THE PARAMETERS OF THE $K^0 - \bar{K}^0$ SYSTEM

A.K.Aringazin
Department of Theoretical Physics
Karaganda State University
Karaganda 470074, Kazakhstan

February 14, 1989

Abstract

Recently, Santilli presented the formalism of Lie-isotopic lifting of the Lorentz group associated with a generalized interval of the form

$$x * x = x^1 b_1^2 x^1 + x^2 b_2^2 x^2 + x^3 b_1^3 x^3 + x^4 c^2 x^4$$

where $b, c = b, c(t, x, V, ...)$. Rectricting b and c to be dependent on the velocity V (Finsler space), we apply the formalism to describe the anomalous energy dependence of the meson lifetimes and some of the fundamental parameters of the K^0-$\bar{K}^0$ system.

Hadronic J. **12** (1989) 71-74.

1 Introduction

In a recent series of papers[1, 2, 3] Santilli put forward a general method, and its various applications, for constructing isotopic generalizations of Lie's theory. In particular, as to Lie-isotopic generalization on metric spaces[2], the following isotopic generalization of the Minkowski interval was studied in detail[1]:

$$x * x = x^1 b_1^2 x^1 + x^2 b_2^2 x^2 + x^3 b_1^3 x^3 + x^4 c^2 x^4, \tag{1}$$

where $b, c = b, c(t, x, V, ...)$. The method provides the form invariance of all possible new metrics (which are considered to be isotopes of the Euclidean one), and is based on a natural generalization of the original Lie symmetry, namely, of Lie algebra, Lie group, enveloping algebra, composition law, etc.[2]. From a geometrical viewpoint, one of the remarkable features of the isotopy is that it provides the universal description of isometries despite the rather different character of the metrics introduced. So, returning to the space-time interval (1), one can specify the metric to the Finslerian case[4, 5, 6], restricting b and c to be dependent on x^i and V^j, where V^j is assumed to be a 4-velocity vector.

On the other hand, there is the intriguing problem of an anomalous energy dependence of the meson lifetimes[7] and of the fundamental parameters of the K^0-$\bar{K}^0$ system[8, 9]. It has been reported by Aronson *et al.*[8] that the data which were obtained from a series of regeneration experiments at Fermilab (in the energy range $E_K = 30 - 130 GeV$) specifically indicate that the values of the mass difference $\Delta m = m_L - m_S$, the lifetime τ_S, and the CP-violation parameters $|\eta_\pm|$ and $\tan\phi_\pm$ as determined in the K^0-$\bar{K}^0$ system rest frame depend on the velocity of this rest frame with respect to the laboratory. Aronson *et al.*[9] arrived at the conclusion that the experimental results, if correct, can not be ascribed to an interaction of kaons with an electromagnetic, hypercharge, or gravitational field, or to the scattering of kaons from stray charges or cosmic neutrinos. In order to describe the anomalous behavior of these four parameters, denoted by x, they introduced the slope parameters $b_x^{(N)}$ defined by

$$x = x_0(1 + b_x^{(N)} \gamma^N), \qquad \gamma = E_K/m, \qquad N = 1, 2 \tag{2}$$

and presented an elaborated analysis of the origin of these $b_x^{(N)}$. We note that Eq.(2) exhibits in fact, up to a factor γ, Blokhintsev-Redei-like behavior as it was described earlier for the lifetimes of unstable particles[10].

Nielsen and Picek[7] have also considered an attractive Lorentz-noninvariant LNI model based on the "minimally" generalized metric

$$g_{ij} = \mathrm{diag}(+1, -1, -1, -1) + \mathrm{diag}(\alpha, \alpha/3, \alpha/3, \alpha/3),$$

which in particular yields the following high-velocity formula for decaying mesons: $\tau = \tau_0\gamma(1 + \frac{4}{3}\alpha\gamma^2)^{-1}$. From a consideration of the experimental results they found the average of the Lorentz-breaking parameter α over the π-, μ-, and K-meson data as follows: $\langle\alpha\rangle = (0.54 \pm 0.17) \times 10^{-3}$.

However, as was outlined by Aronson *et al.*[9] any complete theory must be able to account for the data on $\Delta m = m_L - m_S$ and $|\eta_\pm|$ (not only τ_S). Perhaps, as we shall try to show in this paper, it is the Lie-isotopic Finsler-metric approach (which manifestly exhibits the velocity dependence of the metric, and generalizes, but does not violate, Lorentz invariance) that makes it possible to describe naturally the anomalous Blokhintsev-Redei-like behavior of the parameters of the K^0-$\bar{K}^0$ system at high energies.

The lifting of the Lorentz group for the Lie-isotopic Finsler metric (1) derived in Ref.[1] is used in Sec.2 to demonstrate how the generalized Lie-isotopic Finslerian Lorentz transformations (3) give rise to BR-like behavior of the lifetimes [Eq.(7)].

In Sec.3, we solve the generalized Lie-isotopic wave equation (8) for scalar particles in the metric (1) with $b = b(x, V)$ and $c = \mathrm{const}$, in the WKB approximation. The metric is taken to be PPN expanded to include the function b_3 of the 3-velocity V^a. The kaon wave function (13) proves to be dependent on this function b_3. This leads to energy dependence of the parameter Δm [Eq.(14)]. Ultrarelativistic expansion of b_3 in the factor $\gamma = (1 - V^2/c^2)^{-1/2}$ has been used.

It should be emphasized that the purely gravitational part of the metric (i.e., that associated to the potential $U = r_g/r$ of static weak gravitational field) does not lead to (additional) energy dependence of Δm, in agreement with the statement of Ref.[9] (see Sec.3).

Unfortunately, apart from τ_S and Δm, direct application of the technique considered to the $\eta_\pm$ energy-dependence seems to be misleading. Nevertheless, one can attempt a modification of the basic spinor equation[9] of the K^0-$\bar{K}^0$ system by rewritting it in Lie-isotopic Finslerian covariant form to include effects of V-dependent space-time metric, e.g., of the metric (1). Indeed the analysis of Sec.3 ensures that in the case the metric coefficients

$b_a(V)$ may give rise to V-dependence of the parameter $\eta_{\pm}$ (see also the phenomenological modification of Ref. [9]). However, we shall not consider this problem here.

Also, in Sec.3, we test the predictions for the parameters τ_S and Δm to establish consistency of the results.

The analysis presented in this paper is far from complete. Nevetheless, we hope that the possibility investigated here may have some use in solving the current problem of description of the anomalous energy dependence of the K^0-$\bar{K}^0$ system parameters.

In addition, with the generalized Lorentz transformations (3) the conventional Doppler-shift formula is modified. So, one should inspect the compatibility of the low-velocity limit of the Finslerian Doppler-shift formula with experiment. Accordingly, in the Appendix we make a low-velocity expansion of b_3 [(16)] in the Doppler shift [(15)], arriving at the conclusion that V-dependent terms appear only in second order and higher in V/c.

It should be noted that the Finsler geometry is invoked now to investigate a rather wide range of phenomena, especially in gravitational physics. For a comprehensive introduction to the Finsler geometry, and for a review of its application in modern physics, we refer the reader to Refs.[4, 5, 6] and Refs.[5, 11, 12, 13, 14, 15]

respectively. In particular, post-Newtonian V-dependence effects in Finslerian theory have been studied in Refs.[11, 12, 14, 15], where observational and experimental limits on specific Finslerian parameters were found. Also, for a review of possible V-dependence effects and broken Lorentz symmetry, see Ref. [16].

2 Velocity dependence of the lifetime τ_S

We start with the Lie-isotopic space-time metric defined by (1), which is chosen to be Finslerian, i.e. the coefficients b and c are taken to depend on the coordinates x^i and velocities V^j: $b = b(x, V)$ and $c = c(x, V)$ (i,j,...=1,2,3,4). Using the Lie-isotopic lifting of the Lorentz group, Santilli[1] obtained explicit expressions for generalized Lorentz transformations as follows:

$$z' = \hat{\gamma}(z - Vt), \quad t' = \hat{\gamma}\Big(t - \frac{V b_3^2 z}{c^2}\Big), \tag{3}$$

$$\hat{\gamma} = \Big(1 - \frac{V b_3^2 V}{c^2}\Big)^{-1/2}. \tag{4}$$

(in the limit $b = c = 1$ the usual special relativity theory is recovered). Apparently, Eqs.(3) and (4) lead to a straightforward modification of the conventional lifetime formula, namely,

$$\tau = \tau_0 \hat{\gamma} \tag{5}$$

In the ultrarelativistic approximation, we can write for b_3

$$b_3(x, V) = 1 + \lambda_0 + \lambda_1 \gamma + \lambda_2 \gamma^2 + \cdots, \tag{6}$$

where $\lambda \ll 1$. Here, we have dropped gravitational termms arising from x dependence of b_3, and put, for simplicity, $c = 1$. Combining Eq.(5) with Eq.(6), we obtain the desired BR-like behavior of the lifetimes of mesons in the following form:

$$\tau = \tau_0 \gamma \Bigg[1 + \lambda_0 \gamma^2 + \lambda_1 (1 + \lambda_0) \gamma^3 + \Big(\frac{\lambda_1^2}{2} + \lambda_2 (1 + \lambda_0)\Big) \gamma^4\Bigg]. \tag{7}$$

3 Velocity dependence of the mass difference Δm

As shown in Ref.[9], the difference $\Delta m = m_L - m_S$ comes from that between the phases of free-particle wave functions describing K_L and K_S respectively, and the energy dependence of Δm does not arise from a coupling of the K^0-$\bar{K}^0$ system to an external metric gravitational field. However, when a space-time metric depends manifestly on the velocity V, as it does in the case of Finsler geometry, one must investigate whether it gives rise to an additional V-dependence of the wave functions governed by covariant wave equations.

To this end, we begin with the Lie-isotopic generalization of the Klein-Gordon equation for the scalar particle, which can be written as

$$(D * D - k^2)\phi = 0, \tag{8}$$

where $k = mc/\hbar$ and $D * D$ denotes Lie-isotopic Finslerian lifting of the ordinary contraction, $D * D = D^i g_{ij}(x, V) D^j$. In the case of a static weak gravitational field, we can write a parametrized post-Newtonian (PPN) representation of the Finslerian metric (1) in the form

$$g_{44} = 1 - 2U + O(U^2), \quad g_{4a} = 0,$$

$$g_{ab} = -b_a^2(V)\delta_{ab}(1 + 2\gamma U) + O(U^2), \tag{9}$$

where $U = r_g/r$ and γ is γ_{PPN} (a,b,...=1,2,3). Inserting Eq.(9) in Eq.(8) then gives

$$(1 + 2U)\partial_t^2\phi - (1 - 2\gamma U)\sum_a b_a 2\partial_a^2\phi + (\gamma - 1)\vec{g}\cdot\nabla\phi - k^2\phi = 0, \tag{10}$$

where $\vec{g} = \partial_a U$. Further, in the WKB approximation,

$$\phi(t, z, V) = A\exp(\frac{i}{\hbar}S(z, V)\frac{i}{\hbar}Et)$$

$$S = S_0 + \hbar S_1, \tag{11}$$

Eq.(10) reads

$$\hbar^0 : \ (1 - 2U)E^2 + (1 + 2\gamma U)b_3^2 S_0'^2 - m^2 = 0$$

$$\hbar^1 : \ -(1 - 2\gamma U)(iS_0'' - 2S_0'S_1') + (\gamma - 1)g_z iS_0' = 0 \tag{12}$$

Up to the slowly varying amplitude factor and normalization constant, the solution for ϕ is then given by

$$\phi(t, z, V) \sim \exp\Big(\frac{i}{\hbar}\int^t b_3^{-1}(1 + \gamma_{PPN}U)p'dz' - \frac{i}{\hbar}Et\Big), \tag{13}$$

where $p' = (E'^2 - m^2)^{-1/2}$, $E' = (g_{44})^{-1/2}m/\hat{\gamma}$. So, taking into account the new $\hat{\gamma}$ factor (4), we obtain for the phase difference

$$\Delta\phi = -\frac{i}{\hbar}t_{lab}(\hat{\gamma}b_3^2)^{-1}$$

$$= -\frac{i}{\hbar}t_{lab}\frac{1}{\gamma}\Big[1 - \lambda_0 - \lambda_1\gamma - (\frac{\lambda_0}{2} + \lambda_2)\gamma^2\Big] \tag{14}$$

Two comments are in order: (a) To be strictly correct, we have to note that in Finsler geometry a metric is defined on the tangent bundle $TM \rightarrow M$, with M being a smooth manifold. One can treat a tangent vector $y \in TM_x$

attached at the point $x \in M$ as velocity V of a test particle in physical kinematic context[14, 15]. (b) The result (13) can be obtained in another way since it exhibits in fact space-time properties of the kaon traveling process (see discussion in Ref.[9]).

Note that in our case the single function $b_3(V)$ is used to govern the energy dependence of both the parameters τ_S and Δm. So the question arises as to the consistency of the predictions for the behavior of these parameters in view of the experimental data. First, we note that according to Eqs.(7) and (14), the slope parameters τ_S and Δm have the required opposite signs. Then, the values of the λ's in these equations may be determined to account for the experimental data by identifying corresponding combinations of λ's with the experimental values of $b_x^{(N)}$ presented in Ref.[9].

Appendix

The generalized Lorentz transformations (3) immediately lead to the following Doppler-shift formula:

$$\omega = \omega_0 \frac{1 - V b_3^2 V/c^2}{1 - (V b_3^2/c) \cos \alpha}. \tag{15}$$

Making the general low-velocity approximation

$$b_3^2 = 1 + \kappa_0 + \kappa_1 V + 2 S_a V^a + A_{ab} V^a V^b + O(V^3) + O(U), \tag{16}$$

where κ, S_a, and S_{ab} are Finslerian expansion parameters, in Eq.(15), we obtain

$$\omega = \omega_0 \Big(1 + (1 + \kappa_0) \frac{V}{c} + (1 + \kappa_0 - c\kappa_1) \frac{V^2}{c^2} + 2 S_a V^a \frac{V}{c} \Big) + O(V^3) \tag{17}$$

(we put $\alpha = 0$). According to this equation, V-dependent terms from the expansion (16) enter the second-order Doppler-shift effect.

Acknowledgment

It is pleasure to thank G.S.Asanov for many invaluable discussions, and especially for his helpful hints and criticism.

References

[1] R.M.Santilli, Lett. Nuovo Cimento **37** (1983) 545.

[2] R.M.Santilli, Hadronic J. **8** (1985) 25.

[3] R.M.Santilli, Hadronic J. **8** (1985) 36.

[4] H.Rund, *The differential geometry of Finsler spaces* (Springer, Berlin, 1959).

[5] G.S.Asanov, *Finsler geometry, relativity and gauge theories* (D.Reidel,Dordrecht, 1985).

[6] M.Matsumoto, *Foundations of Finsler geometry and special Finsler spaces* (Kaiseisha Press, Kaiseisha, 1986).

[7] H.B.Nielsen and I.Picek, Phys. Lett. **B144** (1982) 141.

[8] B.H.Aronson, G.J.Bock, H-Y.Cheng, and E.Fischbach, Phys. Rev. **D28** (1983) 476.

[9] B.H.Aronson, G.J.Bock, H-Y.Cheng, and E.Fischbach, Phys. Rev. **D28** (1983) 495.

[10] D.I.Blokhintsev, Phys. Lett. **12** (1964) 272. L.B.Redei, Phys. Rev. **145** (1966) 999.

[11] I.W.Roxburgh and R.K.Tavakol, Gen. Relativ. Gravit. **10** (1979) 307.

[12] A.A.Coley, Gen. Relativ. Gravit. **14** (1982) 1107.

[13] R.K.Tavakol and N. Van den Bergh, Gen. Relativ. Gravit. **18** (1986) 849.

[14] A.K.Aringazin and G.S.Asanov, Gen. Rel. Grav. **17** (1985) 1153.

[15] A.K.Aringazin and G.S.Asanov, Rep. Math. Phys. **25** (1988) 183.

[16] M.Gasperini, Phys. Lett. **B 177** (1986) 51.