NONPOTENTIAL ELASTIC SCATTERING OF SPINNING PARTICLES

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Abstract

In this paper, we consider nonpotential elastic scattering of two spinning particles, within the framework of hadronic mechanics, which is used to account for nonpotential effects.

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1 Introduction

In this paper, we consider nonpotentital elastic scattering of spinning particles. When particles have spins their interactions are in general of noncentral type, i.e., the potential of the interaction depends not only on the relative distance but also on the mutual orientation of spins and on vector of the relative distance, and the same holds for the nonpotential contribution. We consider nonpotetial scattering of particles with spins s_1 and s_2.

Within the framework of non-potential scattering theory[1, 2], we use the isotopically deformed Schrödinger equation, the iso-Schrödinger equation[3], characterized by isotopic operator T is assumed to be responsible for the *non-potential* part of the scattering. In the limit $T \to 1$, the usual potential scattering theory is recovered.

The isotopic lifting of the spin space giving rise to the isotopic spin space[3] is made by the Lie-Santilli theory[4] with the use of operator R acting on spin parts of the wave functions. In the limit $R \to 1$, the usual spin theory is recovered.

In the previous paper[5], we calculated the free iso-propagator in the coordinate and momenta representations, to find the general solution of the iso-Schrödinger equation, the nonpotential scattering amplitude, and represent the solution and amplitude for particular choices of T. The iso-Lippmann-Schwinger equation for the scattering matrix, its general on-shell solution for a separable potential, and non-potential scattering length have been investigated. We presented computations of the (nonpotential) scattering length for the Yamaguchi potential, and particular cases of the isotopic operator $T(k)$.

The present paper relies on the considerations and results of Ref.[5].

The paper is organized as follows.

In Section 2 we develop a formalism to describe T-isotopic (nonpotential) scattering of spinning particle off spinless particle in vacuum.

In Section 3 we generalize the results of Sec.2 to the case of two spinning particles.

In Section 4 we briefly review the eigenvalue problem of spin operators. The generalization of ordinary spin theory is reached via a spin matrix R, which is chosen diagonal.

Section 5 is devoted to T-isotopic scattering problem of spin-half particles.

2 Scattering of spinning particle off spinless particle

Let us consider the scattering of spinning particle off spinless particle.

The potential V entering the Hamiltonian of the system, $H = H_0 + V$, depends both on the relative distance between the particles and on the spin, i.e., V is an operator (matrix) in the spin space (see for example [6]).

Let us take the initial wave function of the system, $\hat{\varphi}_0^+$, in the form of (T-isotopic) product of the (iso-)free wave, characterizing relative motion with momentum k, and spin function,

$$\hat{\varphi}_{0k\mu}^+ = \exp\{ikT(k)r\} * \hat{\chi}_\mu, \tag{1}$$

where $\hat{\chi}_\mu$ is the eigenfunction of the spin operators in isospace[3], $\hat{s}^2$ and $\hat{s}_z$, with μ being 3-projection of spin (see Section 4 for a review).

Then the wave function $\hat{\psi}_{k\mu}^+$ satisfying the iso-Schrödinger equation and describing the scattering can be written as follows:

$$\hat{\psi}_{k\mu}^+(r) = \hat{\psi}_{0k\mu}^+(r) + \int dr'\ \hat{G}^+(r-r') * \hat{V}(r') * \hat{\psi}_{k\mu}^+(r'), \tag{2}$$

or, in more details,

$$\hat{\psi}_{k\mu}^+(r,\sigma) = \hat{\psi}_{0k\mu}^+(r,\sigma) + \sum_{\sigma',\sigma''} \int dr'\ \hat{G}_{\sigma\sigma'}^+(r-r') * \hat{V}_{\sigma',\sigma''}(r') * \hat{\psi}_{k\mu}^+(r',\sigma''), \tag{3}$$

where $\hat{G}_{\sigma\sigma'}^+(r-r')$ is the iso-propagator, which in the coordinate representation has the form

$$\hat{G}_{\sigma\sigma'}^+(r-r') = -\frac{m}{2\pi}\ \frac{\exp\{ikT(k)|r-r'|\}}{|r-r'|} T(k) \sum_{\mu'} \hat{\chi}_{\mu'}(\sigma) R \hat{\chi}_{\mu'}^+(\sigma'), \tag{4}$$

$\hat{V} = VT$ and R is $(2s+1) \times (2s+1)$ matrix.

We assume that $\hat{\chi}_\mu$'s form a complete set of spin functions,

$$\sum_\mu \hat{\chi}_{\mu'}(\sigma) R \hat{\chi}_\mu^+(\sigma') = \hat{\delta}_{\sigma\sigma'} = \hat{I}\delta_{\sigma\sigma'}. \tag{5}$$

According to this, the propagator can be taken in a diagonal form,

$$\hat{G}_{\sigma\sigma'}^+(r-r') = -\frac{m}{2\pi}\ \frac{\exp\{ikT(k)|r-r'|\}}{|r-r'|} T(k) \hat{\delta}_{\sigma\sigma'}. \tag{6}$$

Below, we follow [6] to derive the scattering amplitude and scattering length.

The operator $\hat{J}$ satisfies the equation

$$\hat{V}T(k)\hat{\psi}^{+}_{k\mu} = \hat{J}T(k)\hat{\varphi}^{+}_{0k\mu}. \tag{7}$$

It can be used in equation (2) to write down the initial wave function in the form

$$\hat{\psi}^{+}_{k\mu}(r) = \Big\{\exp[ikTr] + \int dr'\hat{G}^{+}(r-r')T\hat{J}T\exp[ikTr']\Big\}T\hat{\chi}_{\mu}. \tag{8}$$

Expression in curly brackets is an operator in spin space. Using (6) the asymptotics of the wave function (8) can be presented as

$$\hat{\psi}^{+}_{k\mu}(r) \overset{r\to\infty}{\longrightarrow} \Big\{\exp[ikT(k)r] + \frac{\exp[ikT(k)r']}{r}\hat{f}_{\mu}(k,k')\Big\}T(k)\hat{\chi}_{\mu}, \tag{9}$$

where $\hat{f}_{\mu}(k,k')$ is the nonpotential scattering amplitude; k and k' are income and outcome momenta respectively, in iso-Euclidean space with isometric $\hat{\delta} = T\delta$, $\delta = \mathrm{diag}(1,1,1)$. The solution of the associated iso-Lippman-Schwinger equation for the scattering matrix, $\hat{J}$, in this case is similar to that of the spinless case [6]. We do not repeat the derivation here and present the result. Namely, the nonpotential scattering amplitude for the spinning particle is

$$\hat{f}_{\mu}(k,k') = -\frac{m}{2\pi}\,\langle k|T\hat{J}T|k'\rangle = -\frac{m}{2\pi}\,\langle k| * \hat{J} * |k'\rangle \tag{10}$$

or, taking $m = 1/2$,

$$\hat{f}_{\mu}(k,k') = -\frac{\langle k|T\hat{J}T|k'\rangle}{4\pi}. \tag{11}$$

The scattering length, $\hat{a}_{\mu} = -\hat{f}_{\mu}(0,0)$, is then

$$\hat{a}_{\mu} = \frac{\langle 0|T\hat{J}T|0\rangle}{4\pi}. \tag{12}$$

Note that the nonpotential scattering amplitude (11) as well as $\hat{J}$ are operators (matrices) in the isotopic space. The function obtained as the action of the operator $\hat{f}_{\mu}$ on the initial spin function,

$$\hat{\chi}' = \hat{f}_{\mu} * \hat{\chi}_{\mu}. \tag{13}$$

can be considered as the spin function of the particle after the scattering, i.e. the operator $\hat{f}_\mu$ transforms the spin state $\hat{\chi}_\mu$ to the final spin state $\hat{\chi}'$.

The coefficients of the expansion of $\hat{\chi}'$ in terms of the complete set of $\hat{\chi}_\mu$'s,

$$\hat{\chi}' = \sum_{\mu'} \hat{f}_{\mu\mu'} * \hat{\chi}_{\mu'}. \tag{14}$$

are the amplitudes of elastic scattering accompanied by changing of the projection of spin,

$$\hat{f}_{\mu\mu'} = -\frac{\langle k\mu| * \hat{J} * |k'\mu'\rangle}{4\pi}. \tag{15}$$

Here, μ and μ' are projections of spin in the initial and final states respectively.

It should be noted that when the interactions between the particles depend on spin there is no an azimutal symmetry of the scattering. Namely, the (potential or nonpotential) amplitude $\hat{f}_{\mu\mu'}$ depends not only on the scattering angle $\hat{\theta}$ but also on the azimutal angle $\hat{\varphi}$, which is the angle between the plane defining the quantization axis and the plane of the scattering.

Using equation (7), we can define the scattering amplitude in terms of the potential (see Ref.[6] for details),

$$\hat{f}_{\mu\mu'}(\hat{\theta}, \hat{\varphi}) = -\frac{1}{4\pi} \int dr \hat{\chi}^+_{\mu'} R e^{-ik'T(k')r} T(k) V(r) T(k) \hat{\psi}^+_{k\mu}(r), \tag{16}$$

when $\hat{\theta}$ and $\hat{\varphi}$ are isoangles[3].

The differential cross section at fixed values of the projections of spin can then be written

$$\hat{\sigma}(\mu \to \mu') = \tilde{\sigma}(\mu \to \mu')\hat{I} = |\hat{f}_{\mu\mu'}(\hat{\theta}, \hat{\varphi})|^{\hat{2}} |\hat{f}_{\mu\mu'}(\hat{\theta}, \hat{\varphi})| T |\hat{f}_{\mu\mu'}(\hat{\theta}, \hat{\varphi})| \hat{I}, \tag{17}$$

i.e.,

$$\tilde{\sigma}(\mu \to \mu') = |\hat{f}_{\mu\mu'}(\hat{\theta}, \hat{\varphi})|^2, \tag{18}$$

or, in terms of the scattering length,

$$\hat{\sigma}(\mu \to \mu') = 4\pi |\hat{a}_{\mu\mu'}(\hat{\theta}, \hat{\varphi})|^{\hat{2}}. \tag{19}$$

If the projections are not fixed then the cross section (17) should be averaged on the initial projections, and summed over the final projections, namely,

$$\hat{\sigma}(\hat{\theta}) = \frac{1}{2s+1} R \sum_{\mu\mu'} |\hat{f}_{\mu'\mu}(\hat{\theta}, \hat{\varphi})|^{\hat{2}}. \tag{20}$$

Note that since the initial spin projections are averaged, the cross section (20) depends only on the scattering isoangle $\hat{\theta}$.

3 Scattering of spinning particles

In this section, we generalize the results of the previous section to the case when both particles have non-zero spins.

Let $\hat{s}_1$ and $\hat{s}_2$ denote spins of the incoming and target particles, respectively. The initial wave function can be then written as

$$\hat{\varphi}^+_{0k\mu_1\mu_2} = \exp\{ikT(k)r\}T(k)\hat{\chi}_{s_1\mu_1} R \hat{\chi}_{s_2\mu_2}, \tag{21}$$

where μ_1 and μ_2 are projections of spins s_1 and s_2 respectively.

Introduce the wave functions of the channel spin $\hat{\chi}_{s\mu}$ with the help of (T-isotopically lifted) sum rules,

$$\hat{\chi}_{s\mu} = \sum_{\mu_1\mu_2} (s_1\mu_1 s_2\mu_2|s\mu) R \hat{\chi}_{s_1\mu_1} R \hat{\chi}_{s_2\mu_2}. \tag{22}$$

Inverting this relation, we have

$$\hat{\chi}_{s_1\mu_1} R \hat{\chi}_{s_2\mu_2} = \sum_{\mu_1\mu_2} (s_1\mu_1 s_2\mu_2|s\mu) R \hat{\chi}_{s\mu}. \tag{23}$$

Inserting (23) into (22) and noting that the spin function of distinct channels are orthogonal to each other, one can see that the scatterings for the distinct incoming channels are independent of each other. Thus, the scattering of spinning particles can be described in terms of the scattering of spinning particle off spinless particle which has been considered in the previous section.

Namely, the wave function for a *fixed* incoming channel can be written as (cf. eqs. (8),(7),(4))

$$\hat{\psi}^+_{ks\mu}(r) = e^{ikTr} R \hat{\chi}_{s\mu} - \frac{m}{2\pi} \sum_{s'\mu'} \hat{\chi}_{s'\mu'} T(k) \int dr' \frac{\exp[ikT|r-r'|]}{|r-r'|} R \tag{24}$$

$$\times\Big(\hat{\chi}_{s'\mu'}, \hat{V}(r)T\hat{\psi}^+_{ks\mu}(r')\Big).$$

It should be emphasized that the channel spin is not conserved so that in general $s' \neq s$.

Starting with expression (24), we obtain after tedious but straightforward calculations (cf. eq.(16)),

$$\hat{f}_{s'\mu' s\mu}(\hat{\theta},\hat{\varphi}) = -\frac{m}{2\pi}\ T(k)\int dr \hat{\chi}^+_{s'\mu'} Re^{-ik'T(k')r} T(k)\hat{V}(r)T(k)\hat{\psi}^+_{ks\mu}(r). \quad (25)$$

Since the projections of spins are observables while the channel spins are not, the scattering amplitude (25) has no direct physical meaning. However, the observable scattering amplitude, $\hat{f}_{\mu_1'\mu_2'\mu_1\mu_2}$, can be readily expressed in terms of (25) with the use of sum rules (23),

$$\hat{f}_{\mu_1'\mu_2'\mu_1\mu_2}(\hat{\theta},\hat{\varphi}) = \sum_{s\mu s'\mu'} (s_1\mu_1 s_2\mu_2|s\mu)R(s_1'\mu_1's_2'\mu_2'|s'\mu')\hat{f}_{s'\mu' s\mu}(\hat{\theta},\hat{\varphi}). \quad (26)$$

In equation (26) the sum is over all allowed values of the channel spins s and s'.

The generalization of the cross section (19) then reads

$$\tilde{\sigma}(\mu_1\mu_2 \to \mu_1'\mu_2') = |\hat{f}_{\mu_1'\mu_2'\mu_1\mu_2}(\hat{\theta},\hat{\varphi})|^2, \quad (27)$$

or, in terms of the scattering length,

$$\tilde{\sigma}(\mu_1\mu_2 \to \mu_1'\mu_2') = 4\pi|\hat{a}_{\mu_1'\mu_2'\mu_1\mu_2}(\hat{\theta},\hat{\varphi})|^2.$$

Similarly, the generalization of cross section (20) reads

$$\tilde{\sigma}(\hat{\theta}) = \frac{1}{(2s_1+1)(2s_2+1)} R\sum_{s\mu s'\mu'} |\hat{f}_{s\mu s'\mu'}(\hat{\theta},\hat{\varphi})|^2. \quad (28)$$

4 Isoeigenfunctions of spin operators

The wave function $\hat{\psi}^+(r,\sigma)$ of particle with spin s has $(2s+1)$ components, while spin operators are $(2s+1)\times(2s+1)$ matrices. These dimensionalities are preserved under isotopy[3]. Let us denote $\hat{\chi}_{s\mu}$ the eigenfunctions of the operators $\hat{s}^2 = sRs$ and $\hat{s}_z$, (see Chapter 6 of Ref.[3])

$$\hat{s}^2 R\hat{\chi}_{s\mu} = s(s+1)\ R\hat{\chi}_{s\mu}, \qquad \hat{s}_z R\hat{\chi}_{s\mu} = \mu\ R\hat{\chi}_{s\mu}. \quad (29)$$

The matrix R is assumed to be invertible and $\det R > 0$. Note that when R is the usual unit matrix the ordinary eigenvalue problem and notion of spin is recovered.

The functions $\hat{\chi}_{s\mu}$ are orthogonal and normalized due to

$$\sum_s \hat{\chi}_{s\mu'}(\sigma) R \hat{\chi}^+_{s\mu}(\sigma') = \hat{\delta}_{\sigma\sigma'} \equiv \hat{I}_R \delta_{\sigma\sigma'}, \qquad \hat{I}_R = R^{-1}. \tag{30}$$

This is the isotopic generalization of the standard condition of completeness of the set of eigenfunctions.

The representation for which the spin projection has a certain value we have

$$\hat{\chi}_{s\mu}(\sigma) = \hat{\delta}_{\mu,\sigma}. \tag{31}$$

This means that $\hat{\chi}_{s\mu}(\sigma)$ can be presented as $(2s+1)$ column with all elements zero except for one, with $\sigma = \mu$,

$$\hat{\chi}_{s\mu}(\sigma) = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \hat{I}_R \\ \vdots \\ 0 \end{pmatrix} \begin{array}{c} s \\ s-1 \\ \vdots \\ \mu \\ \vdots \\ -s \end{array}, \tag{32}$$

where $\hat{I}_R = R^{-1}$ is the isounit operator; $\hat{I}_R R \hat{\chi} = \hat{\chi}$. Due to this representaion it is easy to check that the $\hat{\chi}_{s\mu}(\sigma)$ form a complete set, i.e.

$$\sum_\mu \hat{\chi}_{s\mu}(\sigma) R \hat{\chi}^+_{s\mu}(\sigma') = \hat{\delta}_{\sigma\sigma'}, \tag{33}$$

and can be considered as a basis in the $(2s+1)$-dimensional spin space. Therefore, any spin state $\hat{\chi}$ can be presented as the linear superposition,

$$\hat{\chi}(\sigma) = \sum_{\mu=-s}^{s} \hat{a}_\mu R \hat{\chi}_\mu(\sigma), \tag{34}$$

where the $\hat{a}_\mu$'s satisfy the expresion

$$\sum_{mu} \hat{a}_\mu R \hat{a}_\mu = \hat{I}_R. \tag{35}$$

5 Isoscattering of spin-half particles

In this section we specify the considerations made in the previous sections to the spin-half case, $s_1 = s_2 = 1/2$ in isospace[3]

From equation (26) we see that the scattering amplitude for two spinning particles can be expressed in terms of the scattering amplitude when one of the particles is spinless. Indeed,

$$\hat{f}_{\mu_1'\mu_2'\mu_1\mu_2}(\hat{\theta},\hat{\varphi}) = \sum_{s\mu s'\mu'} (s_1\mu_1 s_2\mu_2|s\mu) R(s_1'\mu_1' s_2'\mu_2'|s'\mu') T \hat{f}_0(\hat{\theta},\hat{\varphi}) R \hat{\chi}^+_{s'\mu'}. \quad (36)$$

Introducing the notation

$$\hat{C} = \sum_{s\mu s'\mu'} (s_1\mu_1 s_2\mu_2|s\mu) R(s_1'\mu_1' s_2'\mu_2'|s'\mu'), \quad (37)$$

equation (36) can be rewritten

$$\hat{f}_{\mu_1'\mu_2'\mu_1\mu_2}(\hat{\theta},\hat{\varphi}) = \hat{C} \hat{f}_0(\hat{\theta},\hat{\varphi}) R \hat{\chi}^+_{s'\mu'}. \quad (38)$$

where $\hat{f}_0(\hat{\theta},\hat{\varphi})$ is the nonpotential scattering amplitude for spinless particles.

To determing $\hat{f}$ one should find $\hat{C}$ and $\hat{\chi}^+_{s'\mu'}$ while $\hat{f}_0$ has been found in [5].

5.1 Isotopy of Clebch-Gordan coefficients

Introducing the notation

$$\hat{C}_1 = (s_1\mu_1 s_2\mu_2 \hat{|} s\mu) = \hat{C}_{s_1 s_2}(s\mu;\mu_1\mu_2), \quad (39)$$

$$\hat{C}_2 = (s_1'\mu_1' s_2'\mu_2' \hat{|} s'\mu') = \hat{C}_{s_1' s_2'}(s'\mu';\mu_1'\mu_2'),$$

we rewrite (36) in the more convenient form

$$\hat{C} = \sum_{s\mu s'\mu'} \hat{C}_1 * \hat{C}_2 \equiv \sum_{s\mu s'\mu'} \hat{C}_1 R \hat{C}_2. \quad (40)$$

Using the isotopy[3] of Clebch-Gordan coefficients[6] we have

$$\hat{C}_1 = (s_1\mu_1 s_2\mu_2 \hat{|} s\mu) = \langle s_1\mu_1 s_2\mu_2 \hat{|} s\mu\rangle \quad (41)$$

$$\equiv \langle s_1 j_1, s_2 j_2; s_1 \mu_1 s_2 \mu_2 | R | s_1 j_1, s_2 j_2; s\mu \rangle$$

and

$$\hat{C}_2 = (s'_1 \mu'_1 s'_2 \mu'_2 \hat{|} s' \mu') = \langle s'_1 \mu'_1 s'_2 \mu'_2 \hat{|} s' \mu' \rangle$$

$$\equiv \langle s'_1 j'_1, s'_2 j'_2; s'_1 \mu'_1 s'_2 \mu'_2 | R | s'_1 j'_1, s'_2 j'_2; s' \mu' \rangle$$

where

$$R = \mathrm{diag}(g_{kk}) = \pm b^k b_k, \quad g_{kk} > 0, \quad b_k = b_k(t, r, \dot{r}, \ldots), \ k = 1, 2, 3. \tag{42}$$

In equation (40), the sum runs over four indeces, s, μ, s', and μ'. However, we note that since the sum over the primed indeces is the same as the sum over the unprimed indeces, we can perform the sum over s' and μ',

$$\hat{C} = \sum_{s\mu s'\mu'} \hat{C}_1 * \hat{C}_2 \rightarrow \sum_{s\mu} \hat{C}_1 R \hat{C}_2. \tag{43}$$

To rewrite (40) in an explicit form, one needs an explicit form of the orthohonality condition of the iso-Clebch-Gordan coefficients[3, 6],

$$\sum_{\mu_1} \sum_{\mu_2} \hat{C}_{s_1 s_2}(s\mu; \mu - \mu_1 \ \mu_1) g_{\mu s_1 s_2 \mu_1 \mu_2 \mu' s'_1 s'_2 \mu'_1 \mu'_2} \hat{C}_{s'_1 s'_2}(s'\mu'; \mu' - \mu'_1 \ \mu'_1) = \delta_{ss'} g_{kk}, \tag{44}$$

$$\sum_{s_1} \hat{C}_{s_1 s_2}(s\mu; \mu - \mu_1 \ \mu_1) g_{s\mu_1 \mu_2 s_1 s_2 \mu'_1 s'_1 s'_2 \mu'_2 s'} \hat{C}_{s'_1 s'_2}(s'\mu'; \mu' - \mu'_1 \ \mu'_1) = \delta_{\mu\mu'} g_{kk}, \tag{45}$$

$$\sum_{s_2} \hat{C}_{s_1 s_2}(s\mu; \mu - \mu_1 \ \mu_1) g_{s_1 s_2 \mu_2 \mu s \mu' s' s'_1 s'_2 \mu_2} \hat{C}_{s'_1 s'_2}(s'\mu'; \mu' - \mu'_1 \ \mu'_1) = \delta_{\mu_1 \mu'_1} g_{kk},$$

so that

$$\sum_{s_1} \sum_{s_2} \hat{C}_{s_1 s_2}(s\mu; \mu - \mu_1 \ \mu_1) R \hat{C}_{s'_1 s'_2}(s'\mu'; \mu' - \mu'_1 \ \mu'_1) = \delta_{\mu\mu'} \delta_{\mu_1 \mu'_1} g_{kk}. \tag{46}$$

Combining equations (45) and (46) we finally have

$$\sum_{s\mu s'\mu'} \hat{C}_1 R \hat{C}_2 = \delta^2_{ss} \delta^2_{\mu\mu'} \delta^2_{\mu_1 \mu'_1} g^5_{kk}. \tag{47}$$

5.2 Spin-half case

Let us define spin isoeigenfunctions for the case of spin-half particles. For triplet state S_1 there are three symmetric triplet spin functions,

$$\begin{aligned} \hat{\chi}_1^1(1,2) &= \hat{\alpha}(1) * \hat{\alpha}(2), \\ \hat{\chi}_1^0(1,2) &= \frac{1}{\sqrt{2}}[\hat{\alpha}(1) * \hat{\beta}(2) + \hat{\alpha}(2) * \hat{\beta}(1)], \\ \hat{\chi}_1^{-1}(1,2) &= \hat{\beta}(1) * \hat{\beta}(2). \end{aligned} \tag{48}$$

For the singlet state S_0, we have the antisymmetric spin function

$$\hat{\chi}_0^0(1,2) = \frac{1}{\sqrt{2}}[\hat{\alpha}(1) * \hat{\beta}(2) - \hat{\alpha}(2) * \hat{\beta}(1)], \tag{49}$$

where

$$\hat{\alpha} = \hat{\chi}_{1/2}^{1/2} = \begin{pmatrix} g_{11}^{-1/2} \\ 0 \end{pmatrix}, \quad \hat{\beta} = \hat{\chi}_{1/2}^{-1/2} = \begin{pmatrix} 0 \\ g_{22}^{-1/2} \end{pmatrix}. \tag{50}$$

With (50) we have for (48) and (49) explicitly

$$\begin{aligned} \hat{\chi}_1^1(1,2) &= \begin{pmatrix} g_{11}^{-1/2} \\ 0 \end{pmatrix} \begin{pmatrix} g_{11}^{-1/2} \\ 0 \end{pmatrix}, \\ \hat{\chi}_1^0(1,2) &= \frac{1}{\sqrt{2}} \begin{pmatrix} g_{11}^{-1/2} \\ 0 \end{pmatrix}_{(1)} \begin{pmatrix} 0 \\ g_{22}^{-1/2} \end{pmatrix}_{(2)} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ g_{22}^{-1/2} \end{pmatrix}_{(1)} \left(\begin{array}{c} 0 \\ g_{22}^{-1/2} \,) \end{array} \right)_{(2)}, \\ \hat{\chi}_1^{-1}(1,2) &= \begin{pmatrix} 0 \\ g_{22}^{-1/2} \end{pmatrix}_{(1)} \begin{pmatrix} 0 \\ g_{22}^{-1/2} \end{pmatrix}_{(2)}, \end{aligned} \tag{51}$$

$$\hat{\chi}_0^0(1,2) = \frac{1}{\sqrt{2}} \begin{pmatrix} g_{11}^{-1/2} \\ 0 \end{pmatrix}_{(1)} \begin{pmatrix} 0 \\ g_{22}^{-1/2} \end{pmatrix}_{(2)} - \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ g_{22}^{-1/2} \end{pmatrix}_{(1)} \left(\begin{array}{c} 0 \\ g_{22}^{-1/2} \,) \end{array} \right)_{(2)}. \tag{52}$$

Straightforward calculations lead to the following final form of the spin functions:

$$\hat{\chi}_0^1(1,2) = \begin{pmatrix} g_{11}^{-1} \\ 0 \end{pmatrix}, \quad \hat{\chi}_1^{-1}(1,2) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{53}$$

$$\hat{\chi}_1^0(1,2) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ (g_{11}g_{22})^{-1/2} \end{pmatrix},$$

$$\hat{\chi}_0^0(1,2) = \frac{-1}{\sqrt{2}} \begin{pmatrix} 0 \\ (g_{11}g_{22})^{-1/2} \end{pmatrix}. \tag{54}$$

5.3 Scattering amplitudes

By using the results obtained for the spin functions above we can then easily write the triplet and singlet non-potential amplitudes, $\hat{f}(\hat{\theta},\hat{\varphi})_{trpl}$ and $\hat{f}(\hat{\theta},\hat{\varphi})_{sngl}$, respectively.

5.3.1 Triplet and singlet states

The triplet non-potential amplitude ($S = 1$) is

$$\hat{f}(\hat{\theta},\hat{\varphi})_{trpl} = \frac{1}{\sqrt{2}} \begin{pmatrix} (g_{11}^9/g_{22})^{1/2} \\ 0 \end{pmatrix} \delta^2_{ss'}\delta^2_{\mu\mu'}\delta^2_{\mu_1\mu_1'}\hat{f}_0(\hat{\theta},\hat{\varphi}). \tag{55}$$

Singlet non-potential amplitude ($S = 0$) is

$$\hat{f}(\hat{\theta},\hat{\varphi})_{sngl} = \frac{-1}{\sqrt{2}} \begin{pmatrix} (g_{11}^9/g_{22})^{1/2} \\ 0 \end{pmatrix} \delta^2_{ss'}\delta^2_{\mu\mu'}\delta^2_{\mu_1\mu_1'}\hat{f}_0(\hat{\theta},\hat{\varphi}). \tag{56}$$

Comparing the amplitude (56) with (55) we arrive at the conclusion that the triplet and singlet isoscattering amplitudes coincide in *isospace*, i.e.,

$$\hat{f}(\hat{\theta},\hat{\varphi})_{trpl} = -\hat{f}(\hat{\theta},\hat{\varphi})_{sngl}. \tag{57}$$

We see that the non-potential scattering amplitudes for the triplet and singlet states differ only by the overal minus sign, as it is in the usual potential theory. However, we note that this result follows from the choice of diagonal form (42) of the matrix R. Off-diagonal terms in R may spoil such a correspondence between the amplitudes for the triplet and singlet states.

Using the definition of the non-potential scattering length, $\hat{a} = -\hat{f}(0,0)$, we find from (55)-(57),

$$\hat{a}_{trpl} = \frac{1}{\sqrt{2}} \begin{pmatrix} (g_{11}^9/g_{22})^{1/2} \\ 0 \end{pmatrix} \delta^2_{ss'}\delta^2_{\mu\mu'}\delta^2_{\mu_1\mu_1'}\hat{a}_0, \tag{58}$$

$$\hat{a}_{sngl} = \frac{-1}{\sqrt{2}} \begin{pmatrix} (g_{11}^9/g_{22})^{1/2} \\ 0 \end{pmatrix} \delta^2_{ss'} \delta^2_{\mu\mu'} \delta^2_{\mu_1\mu_1'} \hat{a}_0, \tag{59}$$

and

$$\hat{a}_{trpl} = -\hat{a}_{sngl}. \tag{60}$$

5.3.2 Elastic np-scattering

We now present the application of preceding analyzis to the triplet and singlet scattering amplitudes for the elastic np-scattering, with three specific choices of the isotopic element $T(k)$.

The basic formulas are (58)-(60) while the forms of the scattering length $\hat{a}_0$ for a specific $T(k)$ have been calculated in the previous paper[5].

(a) $T(k) = T_0 =$ const.

$$\hat{a}_{trpl} = \frac{1}{\sqrt{2}} \begin{pmatrix} (g_{11}^9/g_{22})^{1/2} \\ 0 \end{pmatrix} \frac{-2(1+\kappa)^2}{\beta(1+\kappa_0 T_0)^2}, \tag{61}$$

$$\hat{a}_{sngl} = -\hat{a}_{trpl}. \tag{62}$$

(b) $T(k) = 1 + \alpha^2 k^2$, $\alpha =$ const.

$$\hat{a}_{trpl} == \frac{1}{\sqrt{2}} \begin{pmatrix} (g_{11}^9/g_{22})^{1/2} \\ 0 \end{pmatrix} \frac{-2(1+\kappa)^2}{\beta(1+\kappa^2)^2(1+\alpha^n)}, \tag{63}$$

$$\hat{a}_{sngl} = -\hat{a}_{trpl}. \tag{64}$$

(c) $T(k) = 1 + \cos(\alpha k)^n$, $\alpha =$ const, $n \geq 2$.

$$\hat{a}_{trpl} == \frac{1}{\sqrt{2}} \begin{pmatrix} (g_{11}^9/g_{22})^{1/2} \\ 0 \end{pmatrix} \frac{-2(1+\kappa)^2}{\beta(1+\kappa^2)^2(1+4\sin(i\alpha)^n)}, \tag{65}$$

$$\hat{a}_{sngl} = -\hat{a}_{trpl}. \tag{66}$$

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